

Soft contribution to $B \rightarrow \gamma \ell \nu_\ell$ and the B -meson distribution amplitude

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Abstract

The $B \rightarrow \gamma \ell \nu_\ell$ decay at large energies of the photon receives a numerically important soft-overlap contribution which is formally of the next-to-leading order in the expansion in the inverse photon energy. We point out that this contribution can be calculated within the framework of heavy-quark expansion and soft-collinear effective theory, making use of dispersion relations and quark-hadron duality. The soft-overlap contribution is obtained in a full analogy with the similar contribution to the $\gamma^* \gamma \rightarrow \pi$ transition form factor. This result strengthens the case for using the $B \rightarrow \gamma \ell \nu_\ell$ decay to constrain the B -meson distribution amplitude and determine its most important parameter, the inverse moment λ_B .

1. The decay $B \rightarrow \gamma \ell \nu_\ell$ at large photon energy E_γ is one of the simplest hadronic processes that can be studied using QCD factorization [1, 2, 3, 4]. This method involves the B -meson light-cone distribution amplitude (DA) [5, 6, 7, 8, 9, 10] as the main nonperturbative input at the leading order in the heavy quark expansion. The $B \rightarrow \gamma \ell \nu_\ell$ decay¹ is therefore best suited for determining the parameters of the B -meson DA and presents a close analog to the photon-pion transition $\gamma^* \gamma \rightarrow \pi$ with one highly-virtual (Q^2) and one real photon, which is used for the determination of the pion DA. The theoretical challenges are also similar, and in both cases are mainly due to the necessity to have a quantitative control over the corrections that are subleading in the expansion parameter, $1/m_b \sim 1/(2E_\gamma)$ or $1/Q^2$, and, in general, are not factorizable. The purpose of this letter is to emphasize this connection and suggest a method to calculate the soft contribution to $B \rightarrow \gamma \ell \nu_\ell$, which closely follows the technique used in $\gamma^* \gamma \rightarrow \pi$.

A recent summary of the theory status of $B \rightarrow \gamma \ell \nu_\ell$ can be found in [11]. The decay amplitude

$$A(B^- \rightarrow \gamma \ell \bar{\nu}_\ell) = \frac{G_F V_{ub}}{\sqrt{2}} \langle \ell \bar{\nu}_\ell \gamma | \bar{\ell} \gamma^\nu (1 - \gamma_5) \nu_\ell \bar{u} \gamma_\nu (1 - \gamma_5) b | B^- \rangle \quad (1)$$

¹to distinguish this important decay channel we suggest to call it *photoleptonic* B -decay.

can be written in terms of the two form factors, F_V and F_A , defined through the Lorentz decomposition of the hadronic tensor

$$\begin{aligned} T_{\mu\nu}(p, q) &= -i \int d^4x e^{ipx} \langle 0 | T \{ j_\mu^{em}(x) \bar{u}(0) \gamma_\nu (1 - \gamma_5) b(0) \} | B^-(p+q) \rangle \\ &= \epsilon_{\mu\nu\tau\rho} p^\tau v^\rho F_V + i [-g_{\mu\nu}(pv) + v_\mu p_\nu] F_A + \dots \end{aligned} \quad (2)$$

Here p and q are the photon and lepton-pair momenta, respectively, so that $(p+q) = m_B v$ is the B -meson momentum in terms of its velocity. In the above, $j_\mu^{em} = \sum_q e_q \bar{q} \gamma_\mu q$ is the electromagnetic current and the ellipses stand for the terms $\sim p_\mu$ and for the contact term. The origin of the latter is explained in [11] (see also [12]). The form factors can be written as functions of the lepton-pair invariant mass squared q^2 , or, equivalently, of the photon energy E_γ in the B -meson rest frame. The differential decay width is given by

$$\frac{d\Gamma}{dE_\gamma} = \frac{\alpha_{em} G_F^2 |V_{ub}|^2}{6\pi^2} m_B E_\gamma^3 \left(1 - \frac{2E_\gamma}{m_B} \right) \left(|F_V|^2 + |\tilde{F}_A|^2 \right), \quad \tilde{F}_A = F_A + \frac{e_\ell f_B}{E_\gamma}, \quad (3)$$

where the contact term is included in the axial form factor, and the lepton mass is neglected.

For large photon energies the form factors can be calculated [11] as

$$\begin{aligned} F_V(E_\gamma) &= \frac{e_u f_B m_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma, \mu) + \left[\xi(E_\gamma) + \frac{e_b f_B m_B}{2E_\gamma m_b} + \frac{e_u f_B m_B}{(2E_\gamma)^2} \right], \\ F_A(E_\gamma) &= \frac{e_u f_B m_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma, \mu) + \left[\xi(E_\gamma) - \frac{e_b f_B m_B}{2E_\gamma m_b} - \frac{e_u f_B m_B}{(2E_\gamma)^2} \right]. \end{aligned} \quad (4)$$

The first term in both expressions represents the leading contribution in the heavy-quark expansion (HQE) that corresponds to the photon emission from the light spectator quark in B meson. In the above, f_B is the decay constant of B meson, and the quantity λ_B is the first inverse moment of the B -meson DA:

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega} \phi_+(\omega, \mu), \quad (5)$$

where the variable ω is the energy of the light-quark in the B -meson (in its rest frame). The factor $R(E_\gamma, \mu)$ in (4) takes into account gluon radiative corrections (see [11] for details) and equals one at the tree level. The scale-dependence of $\phi_+(\omega, \mu)$ was calculated to the leading-logarithmic accuracy in [13].

The terms in square brackets in (4) are the $1/m_b$ and $1/(2E_\gamma)$ power corrections to the leading-order expression. They are written in this particular form to emphasize that some of these corrections are “symmetry-preserving”, i.e. are the same for both form factors F_V and F_A , and some corrections are “symmetry-breaking”, i.e. they are different. The symmetry-preserving corrections parametrized by the function $\xi(E_\gamma)$ present the main difficulty. They are unknown, apart that from the power counting one expects $\xi(E_\gamma) \sim 1/E_\gamma$ with respect to the leading-order term. In the analysis of [11] this function was parametrized as

$$\xi(E_\gamma) = c \frac{f_B}{2E_\gamma}, \quad (6)$$

that is, tacitly assuming a $1/m_B$ suppression with respect to the leading-order term in (4), and the coefficient c was varied between -1 and $+1$.

In this letter we present an approach which allows one to estimate $\xi(E_\gamma)$ to, potentially, 20% accuracy. To be precise, we will be calculating the soft overlap contribution to $\xi(E_\gamma)$ which is not directly accessible in QCD factorization. Generally speaking, there exist also factorizable symmetry-preserving contributions which can be treated in a systematic way within the HQE and added.

2. The main idea which parallels the technique originally suggested in [14] for $\gamma^*\gamma \rightarrow \pi$ transition form factor is to consider the hadronic tensor (2) at negative $p^2 < 0$, $m_B^2 \gg |p^2| \gg \Lambda_{\text{QCD}}^2$, or in other words, an unphysical decay $B \rightarrow \gamma^*\ell\nu_\ell$ involving a (transversely polarized) spacelike photon. The corresponding form factors, now functions of two variables q^2 and p^2 , can be calculated in QCD using HQE and operator-product expansion (OPE) to (at least in principle) arbitrary power accuracy in $1/m_b, 1/E_\gamma, 1/p^2$. The analytic continuation to the real photon limit $p^2 = 0$ can be made using dispersion relation. In this way the explicit evaluation of soft nonfactorizable contributions is effectively replaced by a certain ansatz of the hadronic spectral density in the p^2 -channel.

The starting observation (cf. [14]) is that $F_{V,A}(q^2, p^2)$, the generalized form factors of $B \rightarrow \gamma^*\ell\nu_\ell$, at fixed q^2 satisfy an unsubtracted dispersion relation in the variable p^2 . Separating the contribution of the lowest-lying vector mesons ρ, ω , one can write ($F_{B \rightarrow \gamma^*} = F_V$ or F_A)

$$F_{B \rightarrow \gamma^*}(q^2, p^2) = \frac{f_\rho F_{B \rightarrow \rho}(q^2)}{m_\rho^2 - p^2} + \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\text{Im} F_{B \rightarrow \gamma^*}(q^2, s)}{s - p^2}, \quad (7)$$

where s_0 is a certain effective threshold. Here, we combined the ρ and ω contributions in one resonance term, assuming $m_\rho \simeq m_\omega$ and adopting the zero-width approximation. In the above, f_ρ is the usual decay constant of the vector meson and $F_{B \rightarrow \rho}(q^2)$ is a generic $B \rightarrow \rho(\omega)$ transition form factor. Note that since there are no massless states, the real photon limit is recovered by the simple substitution $p^2 \rightarrow 0$ in (7).

On the other hand, the same form factors can be calculated using HQE. The result satisfies a similar dispersion relation

$$F_{B \rightarrow \gamma^*}^{\text{HQE}}(q^2, p^2) = \frac{1}{\pi} \int_0^{\infty} ds \frac{\text{Im} F_{B \rightarrow \gamma^*}^{\text{HQE}}(q^2, s)}{s - p^2}, \quad (8)$$

where the limit $p^2 \rightarrow 0$ cannot be taken directly: e.g., singular terms in $1/p^2$ appear (cf. [15]), signaling that the HQE cannot be applied to the real photon case $p^2 = 0$ beyond the leading power in $1/m_b$.

The main assumption of the method is that the physical spectral density above the threshold s_0 coincides with the QCD spectral density, as given by the HQE:

$$\text{Im} F_{B \rightarrow \gamma^*}(q^2, s) \simeq \text{Im} F_{B \rightarrow \gamma^*}^{\text{HQE}}(q^2, s) \quad \text{for } s > s_0. \quad (9)$$

This is the usual approximation of local quark-hadron duality. In reality we employ a weaker form of duality, assuming that (9) holds upon averaging with a smooth weight function over a sufficiently broad interval of the variable s .

We expect that the HQE reproduces the “true” form factors $F_{B \rightarrow \gamma^*}(q^2, p^2)$ for asymptotically large values of $-p^2$. Equating the two representations (7) and (8) at $p^2 \rightarrow -\infty$ and subtracting the contributions of $s > s_0$ from both sides one obtains

$$f_\rho F_{B \rightarrow \rho}(q^2) = \frac{1}{\pi} \int_0^{s_0} ds \operatorname{Im} F_{B \rightarrow \gamma^*}^{\text{HQE}}(q^2, s). \quad (10)$$

This relation explains why s_0 is usually referred to as the interval of duality in the vector-meson channel. The perturbatively obtained spectral density in HQE $\operatorname{Im} F_{B \rightarrow \gamma^*}^{\text{HQE}}(q^2, s)$ is a smooth function of s and does not vanish at $s \rightarrow 0$. It is very different from the hadronic spectral density $\operatorname{Im} F_{B \rightarrow \gamma^* \ell \nu_\ell}(q^2, s) \sim \delta(s - m_\rho^2)$. However, the integrals over both spectral densities over a certain region of s coincide; in this sense the QCD description of correlation functions in terms of quark and gluons is dual to the one in terms of hadronic states.

In practical applications of this method one uses an additional device, borrowed from the method of QCD sum rules [16], which allows one to reduce the sensitivity to the duality assumption in (9) and simultaneously suppress the contributions of higher orders in the OPE. This is done going over to the Borel representation $1/(s - p^2) \rightarrow \exp[-s/M^2]$, the net effect being the appearance of an additional weight factor under the integral

$$f_\rho F_{B \rightarrow \rho}(q^2) = \frac{1}{\pi} \int_0^{s_0} ds e^{-(s-m_\rho^2)/M^2} \operatorname{Im} F_{B \rightarrow \gamma^*}^{\text{HQE}}(q^2, s). \quad (11)$$

Varying the Borel parameter within a certain window, usually $M^2 = 1 - 2 \text{ GeV}^2$, one can test the sensitivity of the results to the particular model of the spectral density ².

With this refinement, substituting (11) in (7) and using (9) one obtains for $p^2 \rightarrow 0$ (cf. [14, 15])

$$F_{B \rightarrow \gamma}(q^2) = \frac{1}{\pi} \int_0^{s_0} \frac{ds}{m_\rho^2} \operatorname{Im} F_{B \rightarrow \gamma^*}^{\text{HQE}}(q^2, s) e^{-(s-m_\rho^2)/M^2} + \frac{1}{\pi} \int_{s_0}^{\infty} \frac{ds}{s} \operatorname{Im} F_{B \rightarrow \gamma^*}^{\text{HQE}}(q^2, s). \quad (12)$$

This expression contains two nonperturbative parameters — the vector meson mass m_ρ and effective threshold $s_0 \simeq 1.5 \text{ GeV}^2$ — as compared to the “pure” QCD calculation based on the HQE expansion. The reward is that the HQE can be done to an arbitrary accuracy in the powers of $1/m_b$. The nonfactorizable contributions that are beyond the accuracy of the HQE in the usual treatment are taken into account effectively, via the nonperturbative modification of the spectral density.

3. As an illustration, consider the expression corresponding to the leading-order in HQE diagram of the virtual photon emission from the light spectator-antiquark in B meson. Calculating this diagram in terms of B -meson DA at $p^2 < 0$, we obtain, after replacing $q^2 = m_B^2 - 2m_B E_\gamma + p^2$, the following expression for the form factors defined in (2):

$$F_V^{(0)}(q^2, p^2) = F_{B \rightarrow \gamma^*}^{(0)}(E_\gamma, p^2) = e_u f_B m_B \int_0^\infty d\omega \frac{\phi_+(\omega, \mu)}{2E_\gamma \omega - p^2}, \quad (13)$$

²We note in passing that the relation (11) where the r.h.s. is calculated in terms of HQE and B meson DA's, represents a *light-sone sum rule* for $B \rightarrow \rho$ form factor of the particular type considered in [17, 18].

where we neglected the corrections $\sim \omega/E_\gamma$. In this approximation, the axial form factor $F_A^{(0)}(q^2, p^2)$ coincides with $F_V^{(0)}(q^2, p^2)$. The energy integral in (13) can easily be converted to the form of a dispersion relation by the change of variables $s = 2E_\gamma\omega$. Following the procedure described above and changing the integration variable back to $\omega = s/(2E_\gamma)$, we obtain, at $p^2 \rightarrow 0$,

$$F_{B \rightarrow \gamma}^{(0)}(E_\gamma) = \frac{e_u f_B m_B}{2E_\gamma} \left[(2E_\gamma) \int_0^{s_0/(2E_\gamma)} \frac{d\omega}{m_\rho^2} \phi_+(\omega, \mu) e^{-(2E_\gamma\omega - m_\rho^2)/M^2} + \int_{s_0/(2E_\gamma)}^\infty \frac{d\omega}{\omega} \phi_+(\omega, \mu) \right]. \quad (14)$$

Completing the second integral to run from zero to infinity and subtracting the correction from the first term, we can write this expression as

$$F_{B \rightarrow \gamma}^{(0)}(E_\gamma) = \frac{e_u f_B m_B}{2E_\gamma \lambda_B(\mu)} + \frac{e_u f_B m_B}{2E_\gamma} \int_0^{s_0/(2E_\gamma)} d\omega \left[\frac{2E_\gamma}{m_\rho^2} e^{-(2E_\gamma\omega - m_\rho^2)/M^2} - \frac{1}{\omega} \right] \phi_+(\omega, \mu), \quad (15)$$

where the first term is nothing but the HQE expression for the form factor to leading order in α_s and leading power accuracy in (4). The second term can be identified with the soft correction $\xi(E_\gamma)$ as it appears in the same equations. Note that the modification of the standard HQE expression only concerns the region $\omega < s_0/(2E_\gamma) \sim s_0/m_b$, hence, this is a soft spectator-quark contribution. If $\phi_+(\omega, \mu) \sim \omega$ for $\omega \rightarrow 0$ [5, 7, 8, 9, 10], $\xi^{(0)}(E_\gamma)$ defined by (15) corresponds to a power correction of the order of $s_0/(2E_\gamma)^2$ for $E_\gamma \sim m_b \rightarrow \infty$, in agreement with usual power counting.

We define a rescaled soft contribution $\hat{\xi}(E_\gamma)$ such that

$$F_{B \rightarrow \gamma}(E_\gamma) = \left(\frac{e_u f_B m_B}{2E_\gamma \lambda_B(\mu)} \right) \left(1 + \frac{\hat{\xi}}{2E_\gamma} \right) + \dots, \quad (16)$$

where the expression in the parenthesis is the leading-order result, and we anticipate that the function $\hat{\xi}(E_\gamma)$ depends on E_γ only weakly. The ellipses stand for radiative and “hard” power corrections, cf. (4)³. In the adopted approximation

$$\hat{\xi}^{(0)}(E_\gamma) = 2E_\gamma \lambda_B \int_0^{s_0/(2E_\gamma)} d\omega \left[\frac{2E_\gamma}{m_\rho^2} e^{-(2E_\gamma\omega - m_\rho^2)/M^2} - \frac{1}{\omega} \right] \phi_+(\omega, \mu). \quad (17)$$

Note that the definition of the B -meson DA in the soft-collinear effective theory (SCET) involves a collinear light spectator-antiquark field. If the separation between collinear and soft regions were done with an explicit cutoff, $\omega > \mu_{\text{SC}}^2/E_\gamma$, the leading-order result for the form factors would read

$$F_{B \rightarrow \gamma}^{\text{SCET}}(E_\gamma) = \frac{e_u f_B m_B}{2E_\gamma} \int_{\mu_{\text{SC}}^2/E_\gamma}^\infty \frac{d\omega}{\omega} \phi_+(\omega, \mu). \quad (18)$$

³ E.g., the corrections given by the $\sim e_u$ terms in square brackets in (4) are readily obtained if one retains the $O(\omega/E_\gamma)$ terms in the integral (13).

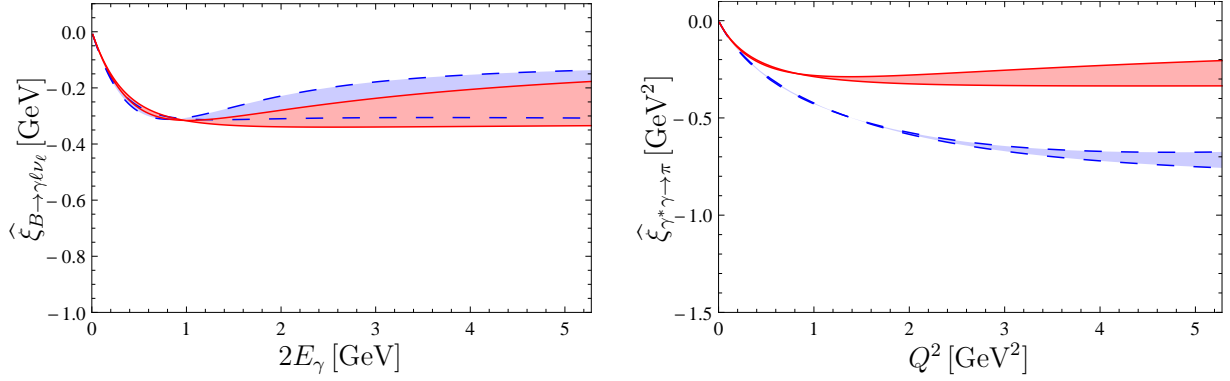


Figure 1: The soft contribution $\hat{\xi}(E_\gamma)$ to the form factors in $B \rightarrow \gamma \ell \nu_\ell$ (left panel) for the first (solid curves) and second (dashed curves) model of the B -meson DA in (19), compared to the soft contribution $\hat{\xi}(Q^2)$ to the $\gamma^* \gamma \rightarrow \pi$ form factor (right panel) for the asymptotic pion DA (solid) and a realistic model with $a_{2,4} \neq 0$ (dashed). The shaded areas correspond to the variation of the Borel parameter in the range $M^2 = 1.0 - 1.5$ GeV².

It is easy to see that the soft correction defined by (14), (15) effectively cuts off the small energy region in a way similar in spirit to (18), with the interval of duality playing the role of the hard cutoff μ_{SC} . We therefore expect that the soft nonperturbative correction is always *negative* relative to the leading-order result because its role is, conceptually, to create a mass gap in the vector-meson mass spectrum.

4. For numerical estimates we will use two models of the B -meson DA:

$$\begin{aligned} \phi_+^I(\omega) &= \frac{\omega}{\lambda_B^2} e^{-\omega/\lambda_B} \quad [5], \\ \phi_+^{II}(\omega) &= \frac{4}{\pi \lambda_B} \frac{\hat{\omega}}{\hat{\omega}^2 + 1} \left[\frac{1}{\hat{\omega}^2 + 1} - \frac{2(\sigma_B - 1)}{\pi^2} \ln \hat{\omega} \right], \quad [8], \end{aligned} \quad (19)$$

where $\hat{\omega} = \omega/1$ GeV and σ_B is the first logarithmic moment $\sigma_B \lambda_B^{-1} = -\int_0^\infty \frac{d\omega}{\omega} \phi_+(\omega, \mu) \ln \frac{\omega}{\mu}$. We take $300 \text{ MeV} < \lambda_B < 600 \text{ MeV}$ and $\sigma_B = 1.5$ [8, 9] (see also [10]) as typical values of the parameters.

The function $\hat{\xi}(E_\gamma)$ calculated for the two models of the B -meson DA assuming the standard choice of the continuum threshold $s_0 = 1.5$ GeV² and $M^2 = 1.0 - 1.5$ GeV² is plotted vs. $2E_\gamma$ in Fig. 1, the left panel. For this plot we have chosen $\lambda_B = 500$ MeV. Note that the curves are essentially flat for $2E_\gamma > 1$ GeV which means that the soft contribution is well described by a power-suppressed correction $\sim 1/(2E_\gamma)$ as compared to the leading-order result.

For comparison, we show in the same figure (right panel) the soft correction to the $\gamma^* \gamma \rightarrow \pi$

transition form factor in the same approximation:

$$\begin{aligned}
Q^2 F_{\gamma^* \gamma \rightarrow \pi}(Q^2) &= \frac{\sqrt{2} f_\pi}{3} \left\{ \int_0^1 \frac{dx}{x} \phi_\pi(x) + \int_0^{x_0} dx \left[\frac{Q^2}{\bar{x} m_\rho^2} e^{(\bar{x} m_\rho^2 - x Q^2)/(\bar{x} M^2)} - \frac{1}{x} \right] \phi_\pi(x) \right\} \\
&\equiv \frac{\sqrt{2} f_\pi}{3} \left(\int_0^1 \frac{dx}{x} \phi_\pi(x) \right) \left[1 + \frac{\hat{\xi}_{\gamma^* \gamma \rightarrow \pi}(Q^2)}{Q^2} \right],
\end{aligned} \tag{20}$$

where $x_0 = s_0/(s_0 + Q^2)$. The results are shown for two sample models of the pion DA: 1) the asymptotic DA $\phi_\pi(x) = 6x(1-x)$ and 2) a realistic DA with nonvanishing Gegenbauer moments $a_2(1\text{GeV}) = 0.17$ and $a_4(1\text{GeV}) = 0.06$. These are typical numbers that are used in the light-cone sum rule analysis of the pion electromagnetic form factor [19] and weak $B \rightarrow \pi \ell \nu_\ell$ decays [20], see also a discussion in [15].

Note that the size of the soft correction to the $\gamma^* \gamma \rightarrow \pi$ form factor for the asymptotic pion DA is very similar to the soft correction to $B \rightarrow \gamma \ell \nu_\ell$ for the existing models (and for $\lambda_B = 500$ MeV). For the realistic pion DA the correction is larger. The difference is partially due to the larger value of the inverse moment $\int (dx/x) \phi_\pi(x) = 3(1 + a_2 + a_4 + \dots)$ playing the same role as λ_B^{-1} for B -meson, but also due to the functional form. If the pion DA has some enhancement close to the end points, as was suspected in particular in view of the BABAR data [21] but also not excluded by Belle [22], then the soft correction is larger and the onset of the asymptotic regime (where it is a $1/Q^4$ correction) occurs much later, see [15, 23].

Varying the inverse moment of the B -meson DA in the interval $300 \text{ MeV} < \lambda_B < 600 \text{ MeV}$ we obtain the following values of the rescaled soft factor $\hat{\xi}$, defined in (16), averaged over the photon energy interval $2 \text{ GeV} < 2E_\gamma < m_B$:

λ_B [MeV]	300	400	500	600
$\hat{\xi}_{\text{Model I}}$ [GeV]	$-(0.50^{+0.04}_{-0.12})$	$-(0.36^{+0.06}_{-0.11})$	$-(0.27^{+0.07}_{-0.09})$	$-(0.22^{+0.07}_{-0.08})$
$\hat{\xi}_{\text{Model II}}$ [GeV]			$-(0.23^{+0.08}_{-0.09})$	

The quoted uncertainties include variations of the Borel mass and the photon energy dependence.

Our result for $\hat{\xi}$ (Model I) translates to the value of the coefficient c in the notation of (6):

$$c = \left(\frac{m_B}{2E_\gamma} \right) \frac{e_u}{\lambda_B} \hat{\xi}^{(0)} = \left(\frac{m_B}{2E_\gamma} \right) \times \begin{cases} -(1.11^{+0.09}_{-0.27}), & \lambda_B = 300 \text{ MeV} \\ -(0.60^{+0.10}_{-0.18}), & \lambda_B = 400 \text{ MeV} \\ -(0.36^{+0.09}_{-0.12}), & \lambda_B = 500 \text{ MeV} \\ -(0.24^{+0.08}_{-0.09}), & \lambda_B = 600 \text{ MeV} \end{cases}, \tag{21}$$

where the uncertainties have the same origin as in the above Table. We emphasize that the soft contribution $\xi(E_\gamma)$ obtained here has a suppression factor $1/(2E_\gamma)$ with respect to the leading term, as compared to the $1/m_B$ suppression assumed in [11]. Note that for large λ_B^{-1} , i.e. for the B -meson DA that is enhanced in the soft region, there is a strong cancellation between

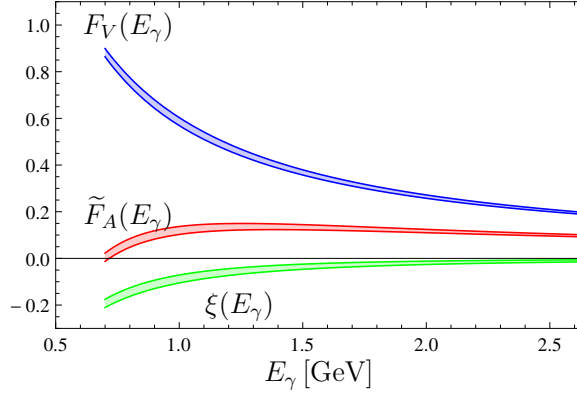


Figure 2: The vector and axial form factors for $B \rightarrow \gamma \ell \nu_\ell$ calculated from (4) at $\lambda_B = 500$ MeV. The lowest curves show the soft-overlap part $\xi(E_\gamma)$ of the form factors. The shaded areas indicate the uncertainties estimated by varying the Borel parameter within $M^2 = 1.0 - 1.5$ GeV² and switching from the model I to model II of the B -meson DA.

the leading term and the soft contribution: In a hypothetical limit $\lambda_B \rightarrow 0$ both terms diverge but the sum of them remains finite.

5. We can employ our estimate of the soft-overlap contribution to calculate the form factors F_V and F_A at large photon energies. To this end we use the expressions in (4) and the result for $R(E_\gamma, \mu)$ given in [11] which includes resummation of radiative corrections to the next-to-leading order logarithmic accuracy. Following [11] we adopt the “soft-collinear” scale $\mu = 1.5$ GeV, the heavy-quark mass scale $m_b = 4.8$ GeV and $f_B = 195$ MeV. In the context of this study we are interested mostly in the uncertainty due to the soft contribution; the variation of the B -meson decay constant within the intervals of the current lattice QCD estimates, will yield an additional (correlated) theoretical uncertainty in both form factors about $\pm 10\%$. The results for the vector and axial form factors — the latter including the contact term — and, separately, for the soft contribution defined in our approximation as $\xi(E_\gamma) = \frac{e_u f_B m_B}{\lambda_B (2E_\gamma)^2} \hat{\zeta}^{(0)}(E_\gamma)$, are shown for the choice $\lambda_B = 500$ MeV in Fig. 2. From (21) it is clear that our estimate of the soft form factor has a smaller error than the interval $-1 < c < 1$ taken in [11].

Using these form factors we calculate the partial branching fraction $BR(B \rightarrow \gamma \ell \nu_\ell)$, integrating (3) over the photon energies $E_{min} < E_\gamma < m_B/2$. The result is shown in Fig. 3 as a function of λ_B for two different choices of the photon energy cut, $E_{min} = 1.0$ (1.7) GeV. For this plot, for definiteness, we take $|V_{ub}| = 3.5 \times 10^{-3}$, in the ballpark of current determinations from exclusive semileptonic B decays. Our main message is that the uncertainty due to the soft overlap contribution is sufficiently small.

The existing experimental data are not yet conclusive. The upper limit on the partial branching fraction $BR(B \rightarrow \gamma \ell \nu_\ell)$ with certain kinematical cuts, including the cut on E_γ , was published by the BABAR collaboration in [24]. This limit is weaker than their previous result quoted in [25]. As explained in [11], the published limit [24] is not yet sufficient to constrain the inverse moment λ_B .

Finally, we suggest to consider the ratio of the photoleptonic and leptonic charged B -meson

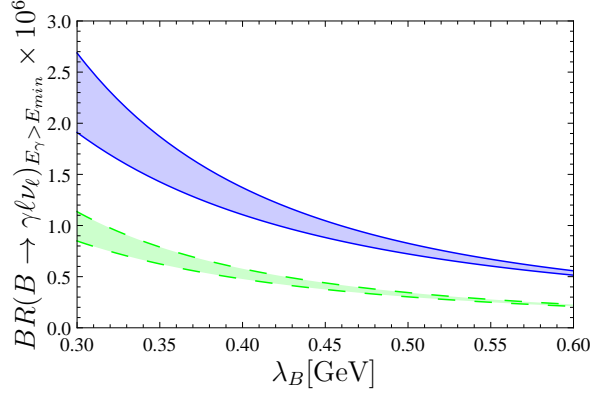


Figure 3: The partial branching fraction $BR(B \rightarrow \gamma \ell \nu_\ell)_{E_\gamma > E_{min}}$ for $E_{min} = 1.0$ GeV (upper, solid) and $E_{min} = 1.7$ GeV (lower, dashed). The uncertainties indicated by shaded areas are of the same origin as in Fig. 2.

decay widths:

$$\begin{aligned}
 R_{\gamma\mu\nu/\tau\nu}(E_{min}) &\equiv \frac{BR(B \rightarrow \gamma \mu \nu_\ell)_{E_\gamma > E_{min}}}{BR(B \rightarrow \tau \nu_\tau)} \\
 &= \frac{4\alpha_{em}}{3\pi m_\tau^2 (1 - m_\tau^2/m_B^2)^2} \int_{E_{min}}^{m_B/2} dE_\gamma E_\gamma^3 \left(1 - \frac{2E_\gamma}{m_B}\right) \left[\frac{|F_V(E_\gamma)|^2 + |\tilde{F}_A(E_\gamma)|^2}{f_B^2} \right], \quad (22)
 \end{aligned}$$

where we neglect the muon mass. Both observables are accessible in the B -factory experiments and their ratio does not depend on V_{ub} and on the B -meson decay constant because f_B enters also the normalization of the form factors. We predict: $R_{\gamma\mu\nu/\tau\nu}(E_{min} = 1.7 \text{ GeV}) = 0.0103, 0.0058, 0.0037, 0.0025$ at $\lambda_B = 300, 400, 500, 600$ MeV, respectively (for model I, $M^2 = 1.0 \text{ GeV}^2$). Note that the recent measurement [26] of $B \rightarrow \tau \nu_\tau$ yields $f_B |V_{ub}| = [7.4 \pm 0.8(stat) \pm 0.5(syst)] \times 10^{-4} \text{ GeV}$, consistent with the lattice-QCD value of f_B and with the value of V_{ub} which we have used above for the estimate of the partial photoleptonic B decay width.

6. To conclude, we have described a method to calculate the soft contribution to the decay $B \rightarrow \gamma \ell \nu_\ell$ which is formally subleading in powers of the photon energy E_γ and is not directly accessible in QCD factorization. This method has originally been developed for another process, the $\gamma^* \gamma \rightarrow \pi$ transition form factor, in which case the QCD calculation is under a better control because the moments of pion DA can be calculated in lattice QCD. The successful description of the experimental data on the $\gamma^* \gamma \rightarrow \pi$ form factor in this framework in the region of momentum transfers $Q^2 = 2 - 6 \text{ GeV}^2$ (see e.g. [15] and references therein) allows one to hope that the same technique will yield sufficiently accurate predictions for the decay $B \rightarrow \gamma \ell \nu_\ell$ as well.

The calculation presented in this letter serves the purpose of demonstration mainly. It can and should be extended in several directions, by replacing the leading-order expression for the spectral density $\text{Im} F_{B \rightarrow \gamma^*}^{\text{HQE}}$ in (12) by the complete HQE expression to the $\mathcal{O}(1/(2E_\gamma), 1/m_B)$ accuracy. First of all, one has to include radiative corrections which give rise to the $R(E_\gamma, \mu)$

factor in (4). This requires a calculation of the terms $\mathcal{O}(\alpha_s)$ in the coefficient function of the B -meson DA for nonzero photon virtualities, which is straightforward. Such corrections can then be resummed using SCET techniques, although we expect that the numerical impact of the resummation will be minimal.

Second, one has to include contributions of higher-twist two-particle and also three-particle B -meson DAs. These contributions contain logarithmic end-point singularities if calculated directly, but give rise to finite contributions to the dispersion integral (12) with the continuum threshold s_0 providing an effective IR cutoff. In other words, the full contribution of twist-four DAs to the form factor is finite, but an attempt to rewrite the answer as a sum of the “pure” HQE expression plus a correction, as in (15) would result in divergent expressions. We believe that the corresponding calculation would be very interesting because the normalization constants in the higher-twist B -meson DAs are related to higher moments of the leading-twist DA, at least within certain schemes used to subtract the corresponding high-energy behavior, see [5, 9, 10, 27].

Third, there are terms related to photon emission at large distances that involve a photon DA. These contributions were estimated for the $\gamma^*\gamma \rightarrow \pi$ transition form factor in [15] in which case they correspond to contributions of *twist-six* operators in the operator product expansion. This example is instructive as it shows that for soft corrections the correspondence between power suppression and twist counting is lost: Contributions of *all* twists to the operator product expansion of the electromagnetic form factors produce power corrections which are suppressed by *one* power of Q^2 with respect to the leading twist result. We expect that the situation with the HQE for the decay $B \rightarrow \gamma \ell \nu_\ell$ is similar.

Last but not least, let us mention that $B \rightarrow \gamma \ell \nu_\ell$ decay amplitudes at large photon energies have also been calculated using light-cone sum rules with photon DA’s and B -meson interpolating current [28, 29, 30]. It would be interesting to clarify the interconnection of this approach with the methods employing the B -meson DA’s in order to gain a more complete picture of the underlying quark-gluon dynamics.

References

- [1] G. P. Korchemsky, D. Pirjol and T. -M. Yan, Phys. Rev. D **61** (2000) 114510.
- [2] S. Descotes-Genon and C. T. Sachrajda, Nucl. Phys. B **650** (2003) 356.
- [3] E. Lunghi, D. Pirjol and D. Wyler, Nucl. Phys. B **649** (2003) 349.
- [4] S. W. Bosch, R. J. Hill, B. O. Lange and M. Neubert, Phys. Rev. D **67** (2003) 094014.
- [5] A. G. Grozin and M. Neubert, Phys. Rev. D **55** (1997) 272.
- [6] M. Beneke and T. Feldmann, Nucl. Phys. B **592** (2001) 3.
- [7] H. Kawamura, J. Kodaira, C. -F. Qiao and K. Tanaka, Phys. Lett. B **523** (2001) 111 [Erratum-ibid. B **536** (2002) 344].
- [8] V. M. Braun, D. Yu. Ivanov and G. P. Korchemsky, Phys. Rev. D **69** (2004) 034014.

- [9] S. J. Lee and M. Neubert, Phys. Rev. D **72** (2005) 094028.
- [10] H. Kawamura and K. Tanaka, Phys. Lett. B **673** (2009) 201.
- [11] M. Beneke and J. Rohrwild, Eur. Phys. J. C **71** (2011) 1818.
- [12] A. Khodjamirian and D. Wyler, In Gurzadyan, V.G. (ed.) et al.: From integrable models to gauge theories* 227-241 [hep-ph/0111249].
- [13] B. O. Lange and M. Neubert, Phys. Rev. Lett. **91** (2003) 102001.
- [14] A. Khodjamirian, Eur. Phys. J. C **6** (1999) 477.
- [15] S. S. Agaev, V. M. Braun, N. Offen and F. A. Porkert, Phys. Rev. D **83** (2011) 054020; arXiv:1206.3968 [hep-ph].
- [16] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **147**, 385 (1979); Nucl. Phys. B **147**, 448 (1979).
- [17] A. Khodjamirian, T. Mannel and N. Offen, Phys. Lett. B **620** (2005) 52; A. Khodjamirian, T. Mannel and N. Offen, Phys. Rev. D **75** (2007) 054013.
- [18] F. De Fazio, T. Feldmann and T. Hurth, Nucl. Phys. B **733** (2006) 1 [Erratum-ibid. B **800** (2008) 405].
- [19] V. M. Braun and I. E. Halperin, Phys. Lett. B **328** (1994) 457; V. M. Braun, A. Khodjamirian and M. Maul, Phys. Rev. D **61** (2000) 073004; J. Bijnens and A. Khodjamirian, Eur. Phys. J. C **26** (2002) 67.
- [20] A. Khodjamirian, T. Mannel, N. Offen and Y. -M. Wang, Phys. Rev. D **83** (2011) 094031.
- [21] B. Aubert *et al.* [The BABAR Collaboration], Phys. Rev. D **80** (2009) 052002.
- [22] S. Uehara *et al.* [Belle Collaboration], arXiv:1205.3249 [hep-ex].
- [23] A. P. Bakulev, S. V. Mikhailov, A. V. Pimikov and N. G. Stefanis, Phys. Rev. D **86** (2012) 031501.
- [24] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **80** (2009) 111105.
- [25] B. Aubert *et al.* [BABAR Collaboration], [arXiv:0704.1478 [hep-ex]].
- [26] I. Adachi *et al.* [Belle Collaboration], arXiv:1208.4678 [hep-ex].
- [27] T. Nishikawa and K. Tanaka, arXiv:1109.6786 [hep-ph].
- [28] A. Khodjamirian, G. Stoll and D. Wyler, Phys. Lett. B **358** (1995) 129,
- [29] G. Eilam, I. E. Halperin and R. R. Mendel, Phys. Lett. B **361** (1995) 137.
- [30] P. Ball and E. Kou, JHEP **0304** (2003) 029.